”On the dependence and the equivalence between algebraic and topological properties of C^* Algebras and their module categories, and combinatorial principles extending ZFC.”

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Abstract

In the recent talk, we try to obtain some combinatorial principles as in, see e.g., [1], from ”seemingly” innocent algebraic or topological properties of C^*- algebras, although the definitions are well-known and actually they strictly relies on ZFC, it turns out that many open questions are simply equivalent to combinatorial ones, which definitely cannot be derived in ZFC.

Moreover, if the ”obvious” implication: Model of ZFC implies ”algebraic” or ”topological property”, respectively. We shall give a long list of such of implications, which hold not only in our category of the main interest, but in general, in much more abstract sence, i.e, for any abelian category of modules (including topological rings and topological modules over them as a particular instance of it), for example, you may find the following references as useful: [2] and [3, 4].

However, what is more difficult, actually, is to establish, if possible, the other direction, namely, which is exactly the combinatorial principle we are dealing with. Equivalently, which is the lattice condition, i.e Boolean Algebraic condition, which can be forced in virtue of [5] to manifest their independence of ZFC.

References

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[4] J. Trlifaj. ”Non-perfect rings and a theorem of Eklof and Shelah.” Commentationes Mathematicae Universitatis Carolinae 32.1, 27-32. 1991.

[5] S. Shelah. Proper and improper forcing. Vol. 5. Cambridge University Press, 2017.